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Time Evolving Theory of the Mechanics of Single Particles and Single Photons

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Abstract

We introduce space-time ensemble methods to formulate definitions of single particles and single photons as local abstractions of constant processes. We find the general form of the corresponding Stueckelberg Lagrangian for Riemannian and Newtonian space-times and supply a physical interpretation for the worldline parameter. We develop the corresponding mechanical theories over the extended configuration space and the extended phase space. We suppose that the background can be represented by an 'external field' and we study several general examples. Certain phenomenological forms do not describe particles, others do not seem to describe theories in which the representation of the background is process independent (Riemannian case). At the canonical level the elimination of second-class constraints associated with null processes generates restrictions on the domain of definition of photon coordinates which correspond to the absence of zero energies. The requirement that the canonical process–anti-process classification exist leads to a factorization condition on the extended phase space which is satisfied for all the cases studied in which the configuration formalisms entail no difficulties, except one, which is the 'minimally' coupled external vector field case over Riemannian space-times. We discuss the observation theoretical significance of our results.

1. *Introduction*

For the history generated by Stueckelberg's action principle (Stueckelberg, 1941, 1942) with the Lagrangian,†

$$L_S = \tfrac{1}{2}(dx/d\lambda)^2 + q(dx/d\lambda).A(x) \tag{1.1}$$

the sign $\epsilon^0 \equiv \epsilon(dx^0/d\lambda)$ is independent of λ, where x^0 is a time coordinate. But for the case,

$$L_S = \tfrac{1}{2}(dx/d\lambda)^2 - a\varphi(x), \qquad a = \text{constant} \tag{1.2}$$

ϵ^0 is not a constant and examples exist in which the curves reverse direction in x^0. In either case, however, Stueckelberg's dynamics is based on a λ-evolution formalism.

† We use the space-favoring metric and the ordinary summation convention; $(dx/d\lambda).A(x)$ means inner product of the vectors $(dx/d\lambda)$ and $A(x)$. Latin indices will run from 1 to 3.

In a modified form, based on the observer time sense (Cawley, 1970) the parameter λ has an interesting physical interpretation whose initial ramifications this paper explores.† Roughly speaking, it is an indexing parameter for actual events, and the role of the space-time formalism based on the Stueckelberg equations is to generate theoretical constructions from the patterns of actual events.

The simplest objects over a space-time S are the local abstractions of the constant processes over S. It seems reasonable to define the elementary particles and photons of S to be these. We do not discuss more general objects in much detail here nor do we study space-times more general than Newtonian and Riemannian. Spacelike 'objects' other than measurements, e.g. tachyons, or possibly more complicated constructions, probably are important for elementary particle physics and maybe for astrophysics as well, so they are important from an observation theoretical point of view. The problem of classifying space-times also is of evident importance; but it seems too ambitious to attempt this now and we restrict our attentions to the most familiar of these.‡

In Section 2 we construct an observation theoretical basis for the Stueckelberg space-time equations. In Section 3 we derive a general form for L_S for a single particle and a single photon, under the definitions of Section 2. In Section 4 we develop the configuration space-time theories in external scalar, vector, and (rank two) tensor fields, and discuss the observer theories briefly. In Section 5 we formulate the canonical theories and in Section 6 we present a discussion based on accumulated results. In Section 7 we make some remarks and in Section 8 we summarize the paper. Finally in an appendix we sketch some important features of a theory of observation based on a space-time approach.

2. *Physical Basis of the Space-time Equations for Localized Objects*

A. *Space-time Ensemble Method*

The point (local) abstraction of a set of experimental conditions expressed in an open statement‡ a is an event, which it must be possible to place into correspondence with any point of a space-time. In a process involving an elementary physical object we assume the observer asserts its actual existence‡ only at discrete instances, $(a_n - e_n)$. He fills in the blanks in that direct knowledge in any single experiment by appeal to an ensemble of other experiments involving duplicate experimental conditions. The

† In the elementary particle physics literature and in some textbooks on mechanics λ is sometimes mistaken for the particle proper time.

‡ Much of the content of this paper is based on physical ideas about the relationship between a physical observer and his methods of observation. We refer to a certain structure based on these as the 'theory of observation' and we discuss its essential features in the appendix. Also we define certain terms there, such as 'process', 'actual occurrence', etc., which are employed in the text.

regularities of the patterns of the events is what guarantees the existence of the classification which defines the ensemble. From these regularities the observer infers the existence of various objects, which are constructions based on the characteristics of the ensemble, at every space-time point intermediate between those of the actual observations.

For example, if we label pairs of events a_n of the same type by values λ_1 and λ_2 of an arbitrary parameter, this construction produces sets of homologous events, $\mathscr{A} = \{a(\lambda)|\lambda \in (\lambda_1, \lambda_2)\}$. Theoretical correspondences of the $\mathscr{A}(\lambda)$ to space-time points $e_\lambda = e(\lambda)$ generate those of the sets. The closures† of the sets are the segments of the world histories of the corresponding graphs.‡

This procedure of theoretical inference based on actual observations defines the notion of a physical object, and through the space-time correspondences grants to the observer the right to represent its history mathematically by a space-time graph. We may say that the existence of an object is implied in the physical sense by the existence of the ensembles involving it, which is the reproducibility of the corresponding experiments.§ Turning the analysis around gives the meaning of this representation: λ, ranging over a suitable point set, is an indexing variable whose boundary values label typical elementary observations.

(It is interesting to observe our answer to the Berkeleyan question: How do we know of an object's existence at space-time points intermediate between those of the actual events, where no one is looking? In the sense given, we know. And rather than being any more artificial than knowledge obtained from actual observations, that sense provides the basis for meaningful representation of those observations. It is the sense of all physical knowledge, in this approach.)

B. *Physical Constructions*

The local abstractions of the constant processes over S correspond to certain of the sets $\mathscr{A}$ of the example above. These are the sets A and $\bar{A}$ obtained from $\mathscr{A}$ by time ordering its points, when this ordering is invariant.‖ Here, up to a sign, λ is an invariant 'causal time' or better 'acausal time' associated with the two kinds of process to whose histories it belongs. We can define a particle (or photon) to be a set A constructed in this way

† In general it might not be necessary to assume that λ is a number or that it is real, though in this paper we do. The topology on the event sets may be defined by the topology of the λ-space. Space-time correspondences of the open sets of events assert theoretical existence of the corresponding objects and so do those of the closed sets. The correspondences of the boundaries also assert actual existence, or actual occurrences of the events.

‡ Up to a possible ordering (see below).

§ We must understand by this that the ensemble represents an entire physical process, that is, a set of actual occurrences of a physical law. We note that this entails no restriction on the character of the object's history.

‖ There are two relative orderings of $\{a(\lambda)\}$ and $\{e(\lambda)\}$, when the points of the latter are identified with space-time coordinates assigned by observation theoretical procedures.

and its anti-particle (anti-photon) as the set $\bar{A}$, which means a particle (photon) of another kind by symmetry since an ordering of λ has no physical basis. The history of a particle (photon) is a future-oriented timelike (null) curve.

The space-time formalisms of the present theory reflect the foregoing distinction between 'noumenal' objects $\mathscr{A}$ and physical objects A, $\bar{A}$. Thus the extended space Lagrangian formalism derived from an observer Lagrangian is a physical space-time formalism, while that deriving from the space-time trajectory Lagrangians† is a noumenal space-time formalism (Section 4, C). The analogous distinction is present also on the extended phase space. In fact, in a space-time theory all the physical constructions result from the classifications produced by noumenal constructions, and the noumenal formalisms form the basis for this procedure.‡

C. *Definition of Particle and Photon*

The primitive ideal constant process over a space-time is its observer. The theoretical possibility of defining the observer as a constant process *in* the presence of his surroundings implies that of the existence of the single particle in these surroundings. So the latter plainly is required in the non-vacuous cases. In addition, over every space-time S, there exist with respect to S two classifications of event pairs. These are the causally ordered and the causally unordered pairs. The theoretical possibility of distinguishing them actually against a given background exists if and only if the single photon can exist theoretically against that background, except for space-times which possess a degenerate notion of simultaneity, such as the Newtonian case, S_N.§

In the theory of observation S_N is distinguished by its degeneracy, which disentangles time coordinate assignments to events from those of the space coordinates. The simplest formal definition of a Newtonian particle requires of the equations whose solutions give its noumenal worldline the existence of an integral having the form

$$f[(dx/d\lambda)^2] = \text{constant} \tag{2.1}$$

where the $x^\mu(\lambda)$ are coordinates of $e(\lambda)$. Here we understand that

$$(dx/d\lambda)^2 = -(dx^0/d\lambda)^2 \tag{2.2}$$

† We mean the 'Stueckelberg-Jacobi' Lagrangians introduced in Cawley (1970).

‡ The word noumenal has metaphysical connotations which are not related to our use of it. The noumenal objects do not exist theoretically nor actually. Nor is this an example of 'absolute' or '*a priori*' existence, but of mathematical existence, which means relative to our theoretical procedure!

§ These observations reveal the reasons for which a particle interpretation to a general time evolving theory is a requirement imposed by a theory of observation, where the basic theoretical assumption is that the observer is physical. In the Schrödinger picture of the quantum theory the particle interpretation to the state vector is its representation as a probability amplitude. In the Heisenberg picture this is provided qualitatively by the indeterminacy principle in the classical phase space.

We disallow explicit x-dependence in equation (2.1) in order to maintain the association of particle and process in its simplest form.† It keeps the scales of x^0 and $|\lambda|$ fixed along the curve. Explicit λ-dependence can be transformed away by a reparametrization.

Evidently we can replace equation (2.1) by

$$\tfrac{1}{2}(dx/d\lambda)^2 = \text{constant} \neq 0 \tag{2.3}$$

where the restriction to nonzero values guarantees that spacelike objects will not be particles. In the Riemannian (metric) space-time framework we replace equation (2.2) by

$$(dx/d\lambda)^2 = g_{\mu\nu}(x)\,(dx^\mu/d\lambda)\,(dx^\nu/d\lambda) \leqslant 0 \tag{2.4}$$

Here we have dropped the restriction to nonzero values which appeared in equation (2.3) because the curves with

$$(dx/d\lambda)^2 = 0 \tag{2.5}$$

are needed for the photons. As there is no causal role for Newtonian null processes, there are no Newtonian 'photons'.‡

We assume that the worldline equations of a single particle and a single photon are generated by a Stueckelberg action principle. For Lagrangians containing no explicit λ-dependence the quantity,

$$R(x, dx/d\lambda) = -p_\lambda = (dx^\mu/d\lambda)\frac{\partial L_S(x, dx/d\lambda)}{\partial(dx^\mu/d\lambda)} - L_S(x, dx/d\lambda) \tag{2.6}$$

is a constant.§ We identify $R(x, dx/d\lambda)$ with the left sides of equations (2.3)–(2.5) and express the requirements embodied therein by writing

$$R(x, dx/d\lambda) = \tfrac{1}{2}(dx/d\lambda)^2 \tag{2.7}$$

So for particles $p_\lambda > 0$ and for photons $p_\lambda = 0$. For both particles and photons we require that ϵ^0 be a constant along the curve, where we understand that x^0 is a time coordinate. The constancy of ϵ^0 is necessary to assure the possibility of the association of particle with process and of photon with null process.

3. *Single Particle and Single Photon Stueckelberg Lagrangians*

To solve equations (2.6) and (2.7) for $L_S(x, dx/d\lambda)$ we write

$$L_S(x, dx/d\lambda) = h((dx/d\lambda)^2) + l_S(x, dx/d\lambda) \tag{3.1}$$

† See Section 6, B for discussion of a formal relaxation of this condition.

‡ Evidently we are using the word photon in a particular sense, one suggested by the special and general theories of relativity. The statement of the text allows for Newtonian theories of light.

§ The arbitrariness of the indexing of events means that the parameter-history which links any two of them has no physical meaning except for that contained in the space-time trajectory. So in the Lagrangian theory for the noumenal worldlines there must exist an indexing parameter λ such that L_s possesses no explicit dependence on it. This guarantees the possibility of passing from a λ-evolution scheme to one based on a space-time variable when the Lagrangian formalism is used.

Substituting (3.1) into (2.6) and using (2.7) we get

$$h((dx/d\lambda)^2) + (dx^\mu/d\lambda)\frac{\partial l_S(x, dx/d\lambda)}{\partial(dx^\mu/d\lambda)} - l_S(x, dx/d\lambda) = \tfrac{1}{2}(dx/d\lambda)^2 \quad (3.2)$$

Choosing

$$h((dx/d\lambda)^2) = \tfrac{1}{2}(dx/d\lambda)^2 \quad (3.3)$$

in equation (3.2) the equation for l_S becomes

$$(dx^\mu/d\lambda)\frac{\partial l_S(x, dx/d\lambda)}{\partial(dx^\mu/d\lambda)} - l_S(x, dx/d\lambda) = 0 \quad (3.4)$$

whose integral surfaces are given by

$$\mathscr{F}(x; l_S^{-1}.dx/d\lambda) = 0 \quad (3.5)$$

where $\mathscr{F}$ is an arbitrary function. Equation (3.5) shows that l_S is a homogeneous first-degree function of the velocities $dx^\mu/d\lambda$; we write

$$l_S(x, dx/d\lambda) = L_S^{(1)}(x, dx/d\lambda) \quad (3.6)$$

and we find that the Stueckelberg Lagrangian for a single particle or a photon must be of the form

$$L_S = \tfrac{1}{2}(dx/d\lambda)^2 + L_S^{(1)}(x, dx/d\lambda) \quad (3.7)$$

4. *Theories of Configuration Particles*

In this section we begin by studying some examples of Stueckelberg equations for particle and photon space-time curves, paying particular attention to external fields and to the constancy of ϵ^0. Then we discuss the observer representations and the extended space noumenal and observer formalisms.

A. *Stueckelberg Level: Constancy of* ϵ^0; *External Fields*

(1) *Riemannian Space-times*

We consider the locally Minkowskian Riemannian space-times, S_R, with symmetric affine connection, wherein for every point e coordinate systems exist in which the metric tensor has the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x), \qquad \text{all } x \quad (4.1\text{a})$$

with

$$h_{\mu\nu} = 0, \qquad h_{\mu\nu,\lambda} = 0, \qquad \text{at } e \quad (4.1\text{b})$$

and where the $\eta_{\mu\nu}$ have the values of the diagonal Minkowski metric tensor. These are the locally inertial (Cartesian) systems for the point e.

From equations (2.6) and (2.7) we have

$$g_{\mu\nu}(dx^\mu/d\lambda)(dx^\nu/d\lambda) = -2p_\lambda = -m^2 \neq 0 \quad (4.2)$$

for particles. The curve generated by L_S cannot reverse direction in time at any point. If we assume it does at the point e_0, and that locally inertial coordinates are chosen with $h_{\mu\nu}(e_0)=0$, then $dx^0/d\lambda=0$ together with equation (4.2) implies

$$\sum_{i=1}^{3}(dx^i/d\lambda)^2=-m^2, \qquad \text{at } e_0 \tag{4.3}$$

and this cannot be satisfied. The classification of the curves on S_R generated by this result, which has been obtained in a geometrical class of coordinate systems, exists in any coordinate system.

But the generalization of equation (2.3) as embodied in equation (2.4) is too great unless the $g_{\mu\nu}$ satisfy conditions which relate local and global properties of the space. Without some restrictions, L_S can generate closed curves timelike at every point, as in the example of the de Sitter space with negative curvature (Synge, 1966). Such a space is not a space-time because all of its points are not ordered partially by processes. Henceforth we assume that S_R is a space-time in our sense. We note that the restrictions on the $g_{\mu\nu}$ might depend upon $L_S^{(1)}$ in general.

For photons $p_\lambda=0$, equation (4.3) gives the vanishing of the remaining first derivatives with respect to λ and we have

$$dx^\mu/d\lambda=0 \tag{4.4}$$

at this point. The situation is more complicated than for particles now and we must distinguish the cases $L_S^{(1)}=0$ and $L_S^{(1)}\neq 0$. In the first instance the Euler–Lagrange equations based on equation (3.7) give

$$\frac{d^2x^\mu}{d\lambda^2}=-g^{\mu\rho}[\rho,\sigma\tau]\frac{dx^\sigma}{d\lambda}\frac{dx^\tau}{d\lambda} \tag{4.5}$$

where $g^{\mu\rho}g_{\rho\nu}=\delta_\nu{}^\mu$, which is the Kronecker δ, and the symmetric three-index symbol is given by

$$[\rho,\sigma\tau]=\tfrac{1}{2}[g_{\rho\sigma,\tau}+g_{\rho\tau,\sigma}-g_{\sigma\tau,\rho}] \tag{4.6}$$

commas denoting differentiation by the indicated variables. Equations (4.4) and (4.5) show that all derivatives of $x^\mu(\lambda)$ vanish at e_0 and the curve therefore degenerates to the uninteresting case of the constant map, into a single space-time point. So again there is no reversal of the sign of $(dx^0/d\lambda)$ at any point of the curve and the corresponding geometrical result follows as before.

If the presence of 'an external vector field'† is represented by equation (1.1), the right side of equation (4.5) contains an additional term proportional to $dx/d\lambda$ and again ϵ^0 is constant for photons. But tensor and scalar fields are less straightforward.

The analogs of (1.1) for tensor and scalar fields respectively have

$$L_S^{(1)}=g|(dx/d\lambda)^2|^{-1/2}(dx^\mu/d\lambda)(dx^\nu/d\lambda)\,G_{\mu\nu}(x) \tag{4.7}$$

$$L_S^{(1)}=f\,|(dx/d\lambda)^2|^{1/2}\,\Phi(x) \tag{4.8}$$

† We consider the meaning of terms such as this in Section 6.

but the variation procedures involving these are not well defined for photons (Cawley, 1970). On the other hand, the Lagrangian given in equation (1.2) does not generate the history of a particle and neither does its tensor analog, which is the one suggested by the Whitehead theory of gravitation, namely

$$L_S = \tfrac{1}{2}(dx/d\lambda)^2 + (dx^\mu/d\lambda)(dx^\nu/d\lambda)\, h^W_{\mu\nu}(x) \tag{4.9}$$

The trouble is that neither has the form of equation (3.7).†

The flat space-time particle trajectory Lagrangians‡ issuing from the 'phenomenological' coupling of (4.7) have the same form as the weak field expansions of those stemming from the Einstein form ($L_S^{(1)} = 0$) with $g_{\mu\nu}$ given by equation (4.1). This is also what results from the flat space-time weak field expansion of the Whitehead form, equation (4.9). So the same lowest-order effects as those produced by (4.7) and (4.9) can be had by taking $L_S^{(1)} = 0$ and choosing $g_{\mu\nu}$ appropriately. Then the effect of any external tensor field§ is represented as a transformation of the space-time metric.

The effect of an external scalar field can be introduced in the same way. Here the two space-times are conformally related. If $g_{\mu\nu}$ is the metric in the absence of the external field and $H_{\mu\nu}$ is the transformed metric, we have

$$H_{\mu\nu}(x) = \sigma(x)^2 g_{\mu\nu}(x) \tag{4.10}$$

where $\sigma(x)$ is the scalar field. To keep the metric signature constant $\sigma(x)$ must be nonvanishing.

The weak field expansions of Lagrangians deriving from (1.2) and (4.8) bear to the Lagrangians generated by (4.10) relations analogous to those existing in the corresponding tensor field cases.

(2) *Newtonian Degenerate Space-time*

The Lagrangian (3.7) with the quadratic term (2.2) gives the space-time trajectory Lagrangians,||

$$L_{SJ} = \mp m x'^0(\alpha) + x'(\alpha) . C(x(\alpha), x'(\alpha)), \qquad m = +(2p_\lambda)^{1/2} \neq 0 \tag{4.11}$$

where α is an arbitrary but definite parameter¶ and primes denote differentiation by α. The sign is $(-\epsilon^0)$ and the C_μ are given by

$$C_\mu(x, x') = \frac{\partial L_S^{(1)}(x, x')}{\partial(x'^\mu)} \tag{4.12}$$

† Nevertheless, the worldlines of point planets and photons do not reverse direction in time for the case in which $h^W_{\mu\nu}$ is determined by a spherically symmetric mass distribution. But see Section 6, B where the analogous scalar field case is discussed. For a lucid exposition of the flat space-time Whitehead theory see J. L. Synge (1951, 1952). A generalization to a class of such theories is given by A. Schild (1956). Whitehead theories on curved space-times were discussed by G. Temple (1924).

‡ See footnote † on p. 104.

§ See footnote on p. 107.

|| By $x' . C$ in equation (4.11) we mean $\sum_{\mu=0}^{3} x'^\mu C_\mu$.

¶ A function of the x^μ.

since the partial derivatives of $L_S^{(1)}(x, dx/d\lambda)$ by the $(dx^\mu/d\lambda)$ must be homogeneous zero degree functions, whence follows their independence of λ'.

The first term of equation (4.11) is an α-derivative and so has no effect on the equations of motion for any given choice of α. Yet it is not supposed to vanish there because of the condition, $m \neq 0$! The source of this peculiar restriction is the defining condition for the noumenal worldline of a particle, in particular the inequality in (2.3) whose purpose is to assure the constancy of ϵ^0. (See also Section 6, A.)

A possible form of $L_S^{(1)}$ is

$$L_S^{(1)} = \tfrac{1}{2}M(d\mathbf{x}/d\lambda)^2(dx^0/d\lambda)^{-1} + q[(d\mathbf{x}/d\lambda).\mathbf{A}(x) - (dx^0/d\lambda)\,V(x)] \tag{4.13}$$

where M and q are constants and where we suppose that $\mathbf{A}(x)$, $V(x)$ transform as representations of the homogeneous Gallilei group (Levy-Leblond, 1967):

$$\mathbf{A}(x) \to R\mathbf{A}(x) \qquad \text{and} \qquad V(x) \to V(x) + \mathbf{v}.R\mathbf{A}(x)$$

where R represents a rotation and $\mathbf{v}$ parametrizes the boost. Then the coefficient of q in equation (4.13) is a Gallilei invariant. Though the first term of equation (4.13) is not invariant, the effect of a space-time coordinate transformation is to add to $L_S^{(1)}$ a λ-derivative, which has no effect on the curve generated by the equations of motion nor on the value of p_λ.

B. *Observer Theories*

The observer Lagrangian is introduced through the observer action (Cawley, 1970), which is an integral over increasing time rather than increasing λ. With $\alpha = x^0 \equiv t$ the two observer Lagrangians corresponding to (4.11) are

$$L_\Omega^{(\pm)} = -m \pm [C_0(\mathbf{x}, \dot{\mathbf{x}}, t) + \dot{x}^k C_k(\mathbf{x}, \dot{\mathbf{x}}, t)], \qquad m \neq 0 \tag{4.14}$$

the dots denoting differentiation by t. With the choice of equation (4.13),

$$L_\Omega^{(\pm)} = -m \pm [\tfrac{1}{2}M\dot{\mathbf{x}}^2 + q(\dot{\mathbf{x}}.A(\mathbf{x}, t) - V(\mathbf{x}, t))] \tag{4.15}$$

Equation (4.15) is strikingly different from the forms resulting from equation (1.1) for special relativistic particles, namely,

$$L_\Omega^{(\pm)} = -m(1 - \dot{\mathbf{x}}^2)^{1/2} \pm q[\dot{\mathbf{x}}.\mathbf{A}(\mathbf{x}, t) - A^0(\mathbf{x}, t)], \qquad m = +(2p_\lambda)^{1/2} \neq 0 \tag{4.16}$$

The equations of motion following from (4.15) are the same for each of the two signs; those following from (4.16) transform into each other under reversal of the sign of q. In the latter the 'rest mass' of the particle is m; in the former the 'inertial mass' of the particle is M. We observe that $m > 0$, but that the sign of M is arbitrary.

For photons there is an additional 'causal energy' coordinate (Cawley, 1970) ω associated with the singular character of the condition $p_\lambda = 0$. Choosing x^0 for the time again, the observer Lagrangians are

$$L_\Omega^{(\pm)} = \tfrac{1}{2}\omega[\gamma_{jk}\dot{x}^j\dot{x}^k - (1-2\gamma_k\dot{x}^k)].(-g_{00}) \pm q[\gamma_{jk}\dot{x}^j A^k - (1-2\gamma_k\dot{x}^k)A^0].(-g_{00}) \quad (4.17)$$

where $\gamma_k = (-g_{00})^{-1}.g_{0k}$ and $\gamma_{jk} = (-g_{00})^{-1}.g_{jk}$. The domain of ω is the positive real axis $(0, \infty)$.

The constancy of ϵ^0 on S_R results in the impossibility of particle speeds

$$v = \left[\sum_{i=1}^{3} (dx^i/dx^0)^2\right]^{1/2}$$

in excess of one in locally inertial frames, anywhere at any time (e_0). For a photon the effect appears as the impossibility of motion in which ω passes through zero, in any coordinate system. These impossibilities are logical ones in the present framework. Space-like objects like tachyons, which transmit energy and momentum, are not particles in the conventional causal framework.†

On S_N no analogous space-time kinematic restrictions on particle motions result from the constancy of ϵ^0.

C. *Extended Space Formalisms*

There are two classes of extended space (configuration space-time) formalisms, that of the noumenal formalism which derives from the two L_{SJ} taken together, and that of the observer formalisms, one for each L_Ω. Examples are the free particle Minkowski configuration observer Lagrangian, which is the same for particle and anti-particle,

$$L_\Omega^{(\pm)\prime} = -m[-x'(\tau)^2]^{1/2} \quad (4.18)$$

and the corresponding noumenal Lagrangian pair

$$L_{\mathcal{N}} = \mp m[-x'(\tau)^2]^{1/2} \quad (4.19)$$

where τ is a free parameter and $x'(\tau) = dx(\tau)/d\tau$, and where we have reset $t = x^0$. The noumenal Lagrangians for particles have the same forms as the trajectory Lagrangians L_{SJ}, but that is not true for photons. The free Minkowski photon noumenal Lagrangians are

$$L_{\mathcal{N}} = \pm\tfrac{1}{2}\omega(x'^0(\tau))^{-1}.(x'(\tau))^2 \quad (4.20)$$

while

$$L_{SJ} = \pm\tfrac{1}{2}\omega.(x'(\alpha))^2 \quad (4.21)$$

All configuration space-time Lagrangians are homogeneous first-degree functions of the τ-derivatives.

† R. G. Cawley (NOL preprint, under revision).

5. *The Factorization Condition and Canonical Particles*

In Section 4, A we pointed out certain problems with naive representations of the effects of external scalar and tensor fields. These disappeared in the approach through the space-time metric. In this section, which constructs a theory of 'canonical particles' as opposed to that of the configuration particles of the previous section, the external vector fields turn out to be troublesome.

A. *Noumenal Canonical Formalisms*

A λ-evolution theory would be a 'Super-causal' theory to be used by a Big Observer whose 'space-time' is five-dimensional. His 'space' would be four-dimensional and would have an indefinite metric. This is the wrong physical interpretation of λ for the present approach, so we don't pursue the matter of a Hamiltonian formulation of the Stueckelberg equations. Canonical formalisms based on the noumenal Lagrangians also give λ-evolution theories, but they can be converted to time evolution theories under certain conditions. Our procedure is to regard the extended phase space as a noumenal space-time and require the existence of the particle and anti-particle classifications in the noumenal canonical formalism. We discuss the canonical formalism first, by means of an example.

For the Newtonian case, corresponding to equations (4.11) and (4.13), the canonical constraints which stem from the extended configuration space noumenal Lagrangian pair are†

$$F(\pm) = p_0 + H^{(\pm)} \approx 0 \tag{5.1}$$

where

$$H^{(\pm)} = H_N(\mathbf{x}, \mathbf{p}, x^0) \pm m \tag{5.2}$$

with‡

$$H_N = (2M)^{-1}(\mathbf{p} - q\mathbf{A})^2 + qV, \qquad M \neq 0 \tag{5.3}$$

and where the p_μ are defined in the usual way from $L_{\mathcal{N}}$. $F(+)$ and $F(-)$ cannot vanish simultaneously since $m \neq 0$ so the surfaces, Σ^+ and Σ^-, defined by equation (5.1) are disjoint. Hence their product also is a constraint, the statement of its vanishing still being a weak equation,

$$F = -F(+)F(-) = -(p_0 + H_N)^2 + m^2 \approx 0 \tag{5.4}$$

Up to multiplication by an arbitrary nonvanishing function u, F is the noumenal total Hamiltonian, which generates the translations in τ.§

† The wavy equals sign denotes weak equality in the sense of Dirac (1950, 1958).

‡ We do not discuss the complicated case $M = 0$ in this paper.

§ Starting from a Lagrangian formalism the restriction $u \neq 0$ is always possible and it assures that the canonical formalism will not be pathological. If u vanished at $\tau = \tau_0$ for example, the Poisson bracket equations would give the vanishing of all the τ-derivatives of all the canonical coordinates at the corresponding point, which would introduce an artificial singularity into the formalism.

The canonical equations of motion based on (5.4) are

$$\begin{aligned} \mathbf{x}' \approx [\mathbf{x}, uF] \approx -2u(p_0 + H_N)\frac{\partial H_N}{\partial \mathbf{p}} &\approx \mp 2mu\frac{\partial H_N}{\partial \mathbf{p}} \\ x^{0\prime} &\approx \mp 2mu \\ \mathbf{p}' &\approx \pm 2mu\frac{\partial H_N}{\partial \mathbf{x}} \\ p_0{}' &\approx \pm 2mu\frac{\partial H_N}{\partial x^0} \end{aligned} \tag{5.5}$$

where we have used equation (5.4) to eliminate the factors of $p_0 + H_N$ from the last members. Particle equations for time evolution may be found by dividing all the equations by the second, a procedure whose possibility depends on the condition $m \neq 0$.

In the general case, the infinitesimal generators of the τ-translations in the particle and anti-particle constraint hypersurfaces Σ^+ and Σ^- are proportional to $F(+)$ and $F(-)$. If the 'τ-evolution' is factorizable into distinct particle and anti-particle components, the change of an arbitrary function of the canonical coordinates induced by the commutator of any two τ-translations, one corresponding to a canonical transformation of the points of Σ^+, the other to a canonical transformation of the points of Σ^-, should vanish. Since the infinitesimal generator of the commutator of two such transformations is proportional to the Poisson bracket of $F(+)$ and $F(-)$, that condition is expressed as

$$\{F(+), F(-)\} = 0 \tag{5.6}$$

The existence of the distinction between the two classes of space-time curve in the canonical formalism is the existence of the classification in that formalism. We take the factorization condition, equation (5.6), as a defining condition to be obeyed by the histories of particles and photons relative to the phase space-time. This is in addition to the conditions leading to $\Sigma^+ \cap \Sigma^- = \varnothing$, which are the defining conditions relative to the configuration space-time. Evidently equation (5.6) is satisfied identically in the Newtonian examples just discussed.

From the Riemannian space-time particle Lagrangians with $L_S^{(1)} = 0$ we get equation (5.1) again. We have

$$H^{(\pm)} = -\gamma^k p_k \pm [(\gamma^k p_k)^2 + \gamma^{jk} p_j p_k + (-g^{00})^{-1} m^2]^{1/2} \tag{5.7}$$

where $\gamma^k = (-g^{00})^{-1} g^{0k}$ and $\gamma^{jk} = (-g^{00})^{-1} g^{jk}$. As before $F(+)$ and $F(-)$ cannot vanish jointly for any space-time point (and arbitrary p_μ) because they cannot do so in the locally inertial coordinates for that point. So the vanishing of (minus) their product is again a constraint,

$$F = (-g^{00})^{-1}(p^2 + m^2) \approx 0 \tag{5.8}$$

and $\Sigma^+ \cap \Sigma^- = \varnothing$.

Up to a multiplicative factor, finite and nonvanishing on $\Sigma^+ \cup \Sigma^-$, the factorization condition, equation (5.6), gives

$$2\gamma^k{}_{,j}(\gamma^j\gamma^l + \gamma^{jl})p_k p_l + (\partial_0 - \gamma^k\partial_k)[(\gamma^l p_l)^2 + \gamma^{lm}p_l p_m + (-g^{00})^{-1}m^2] = 0 \tag{5.9}$$

If Γ is a connected curve in $\Sigma^+ \cup \Sigma^-$, then equation (5.9) is satisfied in Fermi coordinates (Synge, 1966) with respect to Γ, at every point of Γ. Hence it is satisfied in any coordinates, everywhere along Γ. Since Γ is arbitrary there are no connected curves in $\Sigma^+ \cup \Sigma^-$ on which equation (5.6) fails, so the condition is satisfied. Hence all the canonical particle conditions are met.

For photons, with $L_S^{(1)} = 0$, the analysis starts from equation (4.20) and is slightly more complicated. Equation (5.1) results again for the same reasons as before, but now there is an additional primary constraint

$$p_\omega \approx 0 \tag{5.10}$$

To get the forms of $H^{(\pm)}$ we take $\alpha = x^0 = t$ in equation (4.21), getting

$$L_{SJ} = \pm\tfrac{1}{2}\omega(g_{00} + 2g_{0k}v^k + g_{jk}v^j v^k) \tag{5.11}$$

where $v^k = dx^k/dt$, whence

$$\pm\omega^{-1}p_k = g_{0k} + g_{jk}v^j \tag{5.12}$$

The functions $H^{(\pm)}$ are got from

$$H^{(\pm)} = p_k v^k - L_{SJ} = \pm\tfrac{1}{2}\omega(-g_{00} + g_{jk}v^j v^k) \tag{5.13}$$

by solving equations (5.12) for the v^j. Inspecting the expression of the solution as a ratio of determinants we see that it has the form

$$v^j = \pm\omega^{-1}D^j + E^j \tag{5.14}$$

where D^j and E^j are functions of the $g_{\mu\nu}$ and the p_k. Substituting (5.14) into (5.13) we have

$$H^{(\pm)} = \pm a^{(0)}\omega + a^{(1)} \pm a^{(2)}\omega^{-1} \tag{5.15}$$

where

$$a^{(0)} = \tfrac{1}{2}(-g_{00} + g_{ij}E^i E^j), \qquad a^{(1)} = g_{ij}D^i E^j, \qquad a^{(2)} = \tfrac{1}{2}g_{ij}D^i D^j \tag{5.16}$$

so

$$F(\pm) = p_0 + a^{(1)} \pm (a^{(0)}\omega + a^{(2)}\omega^{-1}) \tag{5.17}$$

and the corresponding total Hamiltonians are

$$F_T(\pm) = uF(\pm) + vp_\omega \tag{5.18}$$

with u and v arbitrary functions.

We see by equations (4.1) that for any space-time point $a^{(0)} > 0$ in locally inertial coordinates for that point, and hence in any coordinates. By the same argument $a^{(2)} > 0$ also, unless $p_k = 0$ for all k. The consistency condition $p_\omega' = dp_\omega/d\tau \approx 0$ generates an additional constraint via

$$\{p_\omega, F_T(\pm)\} \approx u\{p_\omega, F(\pm)\} \approx 0$$

namely, by (5.16),

$$-a^{(0)} + \omega^{-2} a^{(2)} \approx 0, \qquad \text{or} \qquad \omega - (a^{(2)}/a^{(0)})^{1/2} \approx 0 \tag{5.19}$$

but no further constraints develop from the remaining consistency conditions.

Since the domain of definition of ω as a coordinate excludes the origin there can be no motion in which all the p_k vanish simultaneously because equation (5.19) then would not be satisfied. Thus the consistency condition implies the existence of an inaccessible region of the x^μ, p_μ part of the extended phase space, defined by the vanishing of the photon's spatial three-momentum. Therefore we may redefine the photon phase space to exclude this region and then set about eliminating the ω, p_ω coordinate pair. This is possible because the constraints (5.10) and (5.19) are second class. Equation (5.18) becomes

$$F_T(\pm) = uF(\pm) \tag{5.20}$$

where

$$F(\pm) = p_0 + a^{(1)} \pm 2(a^{(0)}.a^{(2)})^{1/2} \tag{5.21}$$

and the equations of the eliminated constraints reduce to mere definitions of ω and p_ω in terms of the remaining coordinates x^μ, p_μ.

On the new phase space $F(+)$ and $F(-)$ cannot vanish simultaneously. Moreover, p_0 does not vanish anywhere on $\Sigma^+ \cup \Sigma^-$. To see this, observe that under (5.21) $p_0 = 0$ gives

$$(\mathbf{D}.\mathbf{E})^2 = \mathbf{D}.\mathbf{D} + (\mathbf{D}.\mathbf{D})(\mathbf{E}.\mathbf{E}) \tag{5.22}$$

where the 'scalar' product is defined through the positive form determined by the γ_{jk} [introduced in equation (4.17)]. But equation (5.22) cannot be satisfied since the second term on the right is larger than, or equal to, the left-hand side and the first term is positive. So the four vector p_μ does not vanish on the noumenal constraint hypersurfaces. In addition these are disjoint, and the product of $F(+)$ and $F(-)$ is again a constraint. In time-orthogonal coordinates it is easy to get

$$-F(+)F(-) = F = (-g^{00})^{-1} p^2 \approx 0 \tag{5.23}$$

The factorization condition, equation (5.6), now gives another equation of the type of equation (5.8), namely one which equates to zero a function first degree in the derivatives of the $g_{\mu\nu}$. This can be seen by inspection of equation (5.21) if use is made of the fact that $a^{(1)}$ is a first degree homogeneous function of the p_k, as may be determined from equations (5.12) and (5.14). The same argument as before shows then that the factorization condition is satisfied, so all the canonical photon conditions are met.

B. *External Vector Fields*

Let S be the Minkowski space-time S_M and let L_S be given by equation (1.1). Using the notation of equation (5.1) we find

$$H^{(\pm)} = \pm[(\mathbf{p} - q\mathbf{A})^2 + m^2]^{1/2} + qA^0 \tag{5.24}$$

for particles. Whence

$$\{F(+), F(-)\} = 2\{[(\mathbf{p} - q\mathbf{A})^2 + m^2]^{1/2}, p_0 + qA^0\} \tag{5.25}$$

and equation (5.6) is not satisfied unless

$$q\mathbf{E}.(\mathbf{p} - q\mathbf{A}) = 0 \tag{5.26}$$

where

$$\mathbf{E} = -\partial A^0/\partial \mathbf{x} - \partial \mathbf{A}/\partial x^0 \tag{5.27}$$

Equation (5.27) cannot be satisfied in all inertial coordinate systems unless A^μ is the gradient of a scalar field, but in that case $L_S^{(1)}$ is a λ-derivative and it can be transformed away without affecting the Euler–Lagrange equations. So canonical particles in the presence of a nontrivial external vector field do not exist in the present theory. The same judgment holds against the canonical photons.

C. *Time-evolving Forms of the Canonical Constraints*

In equation (5.1) the coordinate p_0 is conjugate to x^0, modulo $L_{\mathcal{N}}$. This means that for the upper sign the forward time translation generator on the space of x^k, p_k is $H_\Omega^{(+)} = H^{(+)}$, which is determined from $L_\Omega^{(+)}$, while for the lower sign the forward generator $H_\Omega^{(-)}$ is got from $L_\Omega^{(-)}$. The observer representations of the extended phase space constraints corresponding to (5.1) thus are

$$F_\Omega(\pm) = p_0 + H_\Omega^{(\pm)} \approx 0 \tag{5.28}$$

For the free particle on S_M, $F_\Omega(+) = F_\Omega(-)$, so

$$F_\Omega(\pm) = p_0 + H_\Omega^{(\pm)} = p_0 + (\mathbf{p}^2 + m^2)^{1/2} \approx 0 \tag{5.29}$$

while for the case of S_N,

$$H_\Omega^{(\pm)} = m + H_N^{(\pm)} \tag{5.30}$$

where

$$H_N^{(\pm)} = \pm(2M)^{-1}(\mathbf{p} \mp q\mathbf{A})^2 \pm qV \tag{5.31}$$

We observe here that

$$H_\Omega^{(-)}[m; q, M] = H_\Omega^{(+)}[m; -q, -M] \tag{5.32}$$

We remark finally that for a given process on S_N characterized by a value for m, this parameter can be absorbed into p_0 in equation (5.28), by the transformation

$$p_0 \to \overline{p_0} = p_0 + m \tag{5.33}$$

But this transformation does not produce the same effect in equations (5.1) or (5.4).

6. *Discussion*

A. *Geometry and Dynamics*

A dualism of the present theory is the following. On the one hand it deals with noumenal formalisms which derive from classifications of space-time geometrical objects, and on the other hand it deals with time evolving systems of the observer's causal representations. This fact has underlain the approach to mechanics which the present paper has adopted and it has implications for the meaning of the results we have obtained.

(i) The existence of the causal masses, always nonzero and by an arbitrary convention always positive, is the consequence of the existence of the corresponding classifications of timelike physical processes. The same relationship holds between causal energies and null process classifications.

The observer's representation of a process is as a causal unfolding of events. The positive definiteness of ω or constancy of m for a process thus expresses a 'causal conservation law' for the process. If the causal mass of a photon is defined as $m_0 = \epsilon(\omega) = 1$, the law asserts the conservation of causal mass in every constant process.

(ii) A universal (i.e. completely unspecified) Newtonian constant process is generated by the example $L_S^{(1)} = 0$. The space-time curves are timelike, but otherwise arbitrary, and the time evolution of the process is divorced entirely from the motion of the spatial coordinates of the corresponding particles. This result is expressed in the form of the observer Lagrangians,† $L_\Omega^{(+)} = L_\Omega^{(-)} = -m \neq 0$.

The corresponding class of curves on S_R is that of the causal (i.e. timelike or null) geodesics, which is presumably only a subclass of the class of constant process curves. So apparently there is not an analogous 'universal Einstein process'.

B. *Remarks about Generalizing Equation* (2.3)

There is a formal similarity between schemes which start from equations (1.2) and (4.10). Taking $g_{\mu\nu}$ to be the flat metric $\eta_{\mu\nu}$, the former gives

$$L_{SJ} = \mp m_S(-x_\eta'^2)^{1/2}(1 + 2am_S^{-2}\,\varphi(x))^{1/2} \tag{6.1}$$

where the subscript designates the metric, and where $m_S \equiv +(2p_\lambda)^{1/2}$; the latter gives

$$L_{SJ} = \mp m\sigma(x)\,(-x_\eta'^2)^{1/2}, \qquad \sigma > 0 \tag{6.2}$$

† The point of view is quite important to the understanding of a result like this, namely that $L_\Omega^{(\pm)} = -m \neq 0$. If a curve, however arbitrary, *is* given at the level of the noumenal formalism, it is a logical requirement that it not disappear from derived formalism. $L_\Omega^{(\pm)}$ is not 'a Lagrangian' but an observer Lagrangian. While m will show up in the observer action, it seems troublesome that the latter is not Gallilei invariant.

The two Lagrangians are equal if

$$m = m_S \qquad \text{and} \qquad \sigma^2 = 1 + 2m_S^{-2}\, a\varphi \tag{6.3}$$

The condition $\sigma(x) \neq 0$ gives

$$m_S{}^2 + 2a\varphi \neq 0 \tag{6.4}$$

if $m_S \neq 0$, which we recognize as the restriction which prevents the tangent vector from changing its signature along the trajectories of (6.1) since

$$-\tfrac{1}{2}(dx/d\lambda)_\eta{}^2 = p_\lambda + a\varphi(x) \tag{6.5}$$

We note that the left side of (6.4) depends explicitly on the process parameter, m_S.

From equation (6.5) we observe that the flat space-time formalism deriving from (1.2) and satisfying (6.4) may be regarded as based in a generalization of equation (2.3) to the case where the right side is allowed to be a function of x, but which now is required to be nonvanishing. This corresponds to allowing λ, and therefore the propositions a_M, open over S_M, to be functions of the coordinates x^μ of the points of S_M. The realization of such a procedure, which makes the coordinates of S_M physical objects, in this case can be accomplished by introducing another space-time whose processes can supply this physical meaning!

The conformally flat space-time S_R, having $g_{\mu\nu} = \sigma^2 \eta_{\mu\nu}$, serves the purpose, and a conformal regraduation (Walker, 1946; Nariai & Ueno, 1960; Dicke, 1962) defined by

$$(dx/d\lambda)_\eta{}^2 \to \sigma^2(dx/d\lambda)_\eta{}^2 \tag{6.6}$$

where σ is given by (6.3), reduces (6.5) to (2.3). Under the transformation (6.6) the $a_M(x)$ must be mapped into events a_R, whose characterization no longer requires specification of the coordinates.

There is a disadvantage to the S_M theory in the canonical formalism, where we have

$$F(\pm) = p_0 \pm (\mathbf{p}^2 + m_S{}^2 + 2a\varphi)^{1/2} \tag{6.7}$$

By (6.4), $\Sigma^+ \cap \Sigma^- = \varnothing$, but equation (5.6) is not satisfied unless $a\varphi_{,\mu} = 0$ or, barring the trivial case where $a = 0$, unless $\varphi(x)$ is a constant, which is equally trivial.

C. *External Fields and Relativity of Space-times*

A physical process occurs against the time evolving background of the rest of the world, which at bottom the observer must define by the same ensemble methods he uses for all processes. We have not considered the background from this point of view in the present work, but instead have contented ourselves with a model representation, loosely characterized as an external field. The examples of Section 4 serve to illustrate some of that looseness, and in addition we offer the following further observations.

(i) The prototype external field is the one given in equation (4.15), which is supposed to describe a certain class of S_N-theoretical processes. If we regard it as an attempt to represent some given system, then the description it gives represents a 'view from S_N'. A view of this system from S_M, or of another system analogous to it under a suitable test, is provided by equation (4.16). In the mechanics of particles, but for an occasional theoretical aberration, the only other physically interesting example of an external field is that of gravity on S_N, which is described by equation (4.15) as the case $\mathbf{A} = 0$ and $V \propto M$ ($M \neq 0$). So it is enough to focus attention on cases related to that of equation (4.15).

(ii) There are two ways to view the relationship between a given pair of observation theoretical schemes. From the standpoint of 'logical integration',† they are converging or diverging theories. In the cases typified by equations (4.15) and (4.16), for instance, one is the 'nonrelativistic' (i.e., $|\mathbf{v}| \to 0$) limit of the other or the other is a generalization of the one. From the standpoint of 'logical differentiation'† they are parallel structures, parametrized in this case by the value of S, and having distinct physical notions entering the descriptions they represent. Thus the theoretical structure of S_N provides the physical (i.e. space-time geometrical) notions of distance (expressed by Newtonian rigidity of measuring rods), thence of relative velocity, of inertial mass (through the mass dichotomy and the kinetic term proportional to M). On the other hand, with respect to S_M distance as a physical notion is replaced by that of spacelike interval (proper length) and velocity by two-way velocity (twin paradox). The distinction of m and M gives way to the single notion of the causal (or 'rest') mass, m.

(iii) Logical integration is used to secure the physical interpretation of phenomena in the general theory of relativity, where the curvature of the metric is represented as the 'effect' (sometimes 'cause'!) of 'external gravitational fields' in the tangent spaces, S_M. This is accomplished by identifications which rely upon interpretation of the terms of a Lagrangian decomposed into a 'kinetic' part like the first term of (4.16) and an 'external field' part like the second; e.g. equations (4.1a), (4.7), or (4.9). These in turn rely upon a program for logical integration between S_M and S_N.

In the usual attempts at theoretical extrapolation the starting point involves a selection of observation theoretical notions which exist for the context of the limiting theory. For example, the physical notions of gravitational and inertial mass, defined by the correspondences to objects and processes of S_N, are used to express the principle of equivalence. Under the logical differentiation which attends the appearance of a new observation (i.e. space-time or matter) theoretical framework, such physical notions may become altered or even disappear. Thus, viewed from S_R, the principle of equivalence seems meaningless owing to the fact that the notion of

† These terms are drawn from the language of Tisza's (1966) 'logical analysis'.

a gravitational field is replaced by a space-time geometrical structure effect.†

It is reasonable to expect other effects of logical differentiation, in particular such as may involve external fields and background like that just discussed between S_N and S_R. In the representation given by equation (4.9) of an alleged external field on S_M the classifications of the curves are not those of particles, that is λ depends on x. In the case of equation (4.7) the field is not external, in this sense: that for processes the Stueckelberg action principle and L_S have explicit p_λ-dependence, being undefined for those processes having $p_\lambda = 0$. Neither of these features is present in the approach to the space-time classification problem that proceeds through the metric.

(iv) It is in the consideration of spacelike phenomena, namely forces, that one is led to the external field representation of the background on S_N. As far as we know there is no comparable theoretical basis for the external field representation over any Riemannian space-time, including S_M. The problem is the lack of a degenerate simultaneity and the apparent consequent absence of an appropriate nonlocal version of Newton's Third Law. It is in fact for precisely this reason that the notion of force in the special and general theories of relativity generally has been abandoned in favor of the external field idea, and the S_N theoretical structure obviously is the starting point for this.

Evidently these procedures represent extrapolation attempts from the limiting S_N theories, and the theories which have been obtained historically by this procedure are not unique. Thus for gravitational phenomena the external tensor field gave way to the S_R theory while for electromagnetism it gave the Maxwell vector field theory on S_M. For theories of configuration particles the process and anti-process classifications factor in each of these cases while for canonical particles in the latter case they do not, according to the result of Section 5, B. In the spirit of the discussion presented in the rest of this section, this result presumably reflects a logical differentiation effect which attends the passage from the S_M configuration theory to the S_M canonical theory in the case of the external vector field representation of the background. No analogous differentiation effects are reflected in the corresponding pair of S_N theories.

(v) In the Riemannian examples we have discussed where the noumenal Lagrangian particle formalism produces a noumenal canonical particle formalism, which, modulo a gauge transformation, is the vanishing vector field case, the causal mass disappears from the configuration equations of motion. But it does not disappear from the corresponding canonical formalism as equations (5.1), (5.7) and (5.8) show. The same is true for the S_N formalisms. For photons, when the causal energy coordinate ω is removed from the canonical formalism, the restriction which appears on

† So we add our voice to the theoretical din concerning this point: The assertion that the principle of equivalence is meaningless in the context of S_R may or may not be right. Its observation theoretical meaning is well defined over S_N however, and the physical existence of inertial masses and gravitational fields *is* assured by the actual data!

the domain of definition of the momentum variables produces a universal unit null causal mass, $m_0 = |p_0(\mathbf{p}^2)^{-1/2}| = 1$, which again is an object present in the canonical formalism.

7. *Remarks*

The vector fields are extremely important in physics and also have been persistent troublemakers. The extraordinary theoretical circumstance represented by quantum electrodynamics may be an example of this. Gauge theories, full of ubiquitous theoretical problems of one kind or another, but which almost certainly work well(!) in some sense have not been confined to the case of electromagnetism. These problems deserve and get considerable attention and in the present approach we have uncovered what looks like another one. The last sentences of Section 6, C (iv) identify this as the problem of finding a canonical particle theory on S_M (or S_R) whose configuration theory on S_M (or S_R), in some limit, is described in terms of coupling to a background vector field.†

The difference between corresponding S_N and S_M canonical theories seems curious in this regard. We are reminded of the discovery of Jauch concerning gauge invariance and Gallilei invariance in the Hilbert space framework of quantum theory (Jauch, 1964). We may speculate that there is some connection between this result, the observation of Section 6, C (v), and the business about causal and inertial masses between S_N and S_M [Sections 6, C (i) and (ii)], but at the moment we do not know what this might be.

Finally we remark that the present formalism is "observation theoretical" rather than 'classical'.

8. *Summary*

We have developed the theories of the single particle and the single photon as local abstractions of physical processes over Newtonian and Riemannian space-times, S_N and S_R. The observation theoretical formalisms are constructed by space-time ensemble methods. To every noumenal particle (photon) classification there corresponds two particle (photon) types, defined by the two possibilities for the ordering of the noumenal event labelling index λ relative to the causal ordering of the time coordinates.

The existence requirement for the same classifications in the noumenal canonical formalisms also includes a factorization condition on the extended phase space. The condition is not satisfied in this case for particles or photons in an external vector field on S_R, but it is satisfied if $L_S^{(1)} = 0$. It is always satisfied on S_N. The domain of definition of photon momentum coordinates is restricted by these existence requirements to exclude zero spatial momentum and zero energy.

† The results of Section 5 hold for S_R in general and in addition entail no assumptions about field equations for $A^\mu(x)$.

The observer representations of these conditions are that no particle goes faster than light in locally inertial frames on S_R and no photon motion with vanishing (causal) energy is possible. In the present scheme these are logical requirements, rather than physical requirements. On S_N there are no corresponding observer representations of these restrictions in the particle equations of motion.

The causal mass is conserved in every constant process. On S_N, the causal and inertial masses are distinct. In the absence of external vector fields on S_R the causal mass disappears from the configuration equations of motion in the treatment of external scalar and tensor fields through the metric, but it appears in the corresponding canonical formalisms. The same is true for the examples studied on S_N, although m can be transformed away in the time-evolving canonical theory of the observer.

Finally we have not considered curved space-times tangent to S_N, but we think it would be worthwhile to do this (Havas, 1964). In addition it would be interesting to work out the S_N-theories of the $M = 0$ particle. And in the case of S_R we have not considered questions which might arise when there are singularities, either in the coordinates or in the geometry (respectively, at the Schwarzschild radius or at $r = 0$ in the example provided by the Schwarzschild geometry).

Appendix

The office of a theory of observation is to represent those procedures of physical observers which determine the essential structure of physical theory. Our effort to construct such a theory is unfinished, but its broadest outlines, which we summarize here, seem clear. We already have discussed in some detail the physical motivation for the approach, which is to identify an absolute causal ordering with an invariant time ordering in the context of a conventional space-time framework.† The formalism which appears to be most natural for these purposes is that of mathematical logic.

We use designatory expressions or open propositions a for theoretical descriptions of experimental conditions. The logical variable e takes values which are domains or points of a space-time S, and the propositions $(a - e)$ assert correspondences of the conditions to the domains. We identify the actual (observed) existence or occurrence at e of conditions described in a with the truth of the proposition $(a - e)$.

We postulate an absolute invariant cause-effect ordering defined so as to coincide with an invariant overall time ordering. A physical law is an invariant causally ordered pair of open propositions, $a \Rightarrow b$, the cause a 'formally implying' the effect b. We identify the actual (observed) occurrence at the ordered pair (e_1, e_2) of the phenomenon represented in $a \Rightarrow b$ with the truth of the correspondence proposition, $[a \Rightarrow b - (e_1, e_2)]$. It is necessary to prescribe conditions for the latter; in accordance with the above postulate, one of these is that e_1 be earlier than e_2 in the geometry

† See footnote † on p. 110.

of S. The spacelike separated occurrences are unordered causally, by definition of causal. A process is a causally ordered chain of occurrences, and for a constant process the conditions in the chain are the same, the law having the form $a \Rightarrow a$.

We assume that physical objects, whose names are used to express the a's, are defined from ensembles of complete experiments, the latter being sets of correspondence propositions. The fundamental observation theoretical problem then is to decide upon the correlations which the ensemble of experiments provides, and the causal postulate rests on the assumption that this can be done always in such a way as to assure the coincidence of the observer's causal representation with the invariant overall time ordering.

In a scheme of this kind the physical existence of objects and occurrence of phenomena and events receive theoretical definitions, namely relative to the observation procedures the theory describes to be those of a physical observer. In this sense, for example, a particle 'going backwards' in time could not exist physically. The results of the present paper attain their significance in this observation theoretical context, and in that of our assumption that a theory of this kind can be constructed which is free from contradictions.

We propose to regard such a theory as providing a framework for the accommodation of fundamental physical theories as world models. One such model for example depicts matter in terms of particles on Newtonian space-time.

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